A Field Study on the Aweraness of Emotional Labour: Kocaeli Case

Eiji Okano [1] Masataka Eguchi [2]

Abstract

We develop a class of DSGE models with nominal rigidities and investigate importance of awareness of default risk at the point of conducting monetary and fiscal policies. We introduce the default risk in the model. The default mechanism is based on Uribe's who show the trade-off between stabilizing inflation and suppressing default. We assume two policy authorities: the policy authority who are aware of the default risk and it who are not. If the policy authorities are aware of the default risk, the welfare costs function which stems from second- order approximated utility function contains the quadratic term of the premium difference which is difference between the (virtual) government debt yield and its coupon rate. If they are not, the welfare costs function does not contain such a quadratic term. We analyze two policies: the exact and the false policies. The exact is optimal monetary and fiscal policy and conducted by the policy authorities which are aware of default risk. The false is optimal monetary and fiscal policy and conducted by the policy authorities which are not aware of default risk. We solve the LQ problem and the model are solved numerically. Further, we calculate coefficients of simple rules which are a class of Taylor rules and a class of Bohn rule. We find that there is no distinction on simple rules between the exact and false policies if the interest spread is low. However, if the interest spread in the steady state is high, the policy authorities should give up to stabilize inflation and minimize the premium difference. We calculate welfare costs under two policies. if the interest spread is low, there is much difference on welfare costs between two policies. However, if it is high, the difference on the welfare costs are not negligible and is very high. If policy authorities are not aware of the default risk, the policy authorities introduce tighter policy which generates much welfare costs. If the interest spread is high, policy authorities should be aware of the default risk.

Keywords: Sovereign Risk; Optimal Monetary Policy; Fiscal Theory of the Price Level

JEL Codes : E52; E60

[1] **Corresponding Author:** Nagoya City University Birkbeck, University of London. eiji okano@econ.nagoya-cu.ac.jp
[2] Komazawa University.

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Preliminary. Comments are Welcome.

Importance of Awareness of Default Risk on Conducting Monetary and Fiscal Policies *

OKANO, Eiji†
Nagoya City University
Birkbeck, University of London

Masataka Eguchi
Komazawa University

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We develop a class of DSGE models with nominal rigidities and investigate importance of awareness of default risk at the point of conducting monetary and fiscal policies. We introduce the default risk in the model. The default mechanism is based on Uribe's who show the trade-off between stabilizing inflation and suppressing default. We assume two policy authorities: the policy authority who are aware of the default risk and it who are not. If the policy authorities are aware of the default risk, the welfare costs function which stems from second-order approximated utility function contains the quadratic term of the premium difference which is difference between the (virtual) government debt yield and its coupon rate. If they are not, the welfare costs function does not contain such a quadratic term. We analyze two policies: the exact and the false policies. The exact is optimal monetary and fiscal policy and conducted by the policy authorities which are aware of default risk. The false is optimal monetary and fiscal policy and conducted by the policy authorities which are not aware of default risk. We solve the LQ problem and the model are solved numerically. Further, we calculate coefficients of simple rules which are a class of Taylor rules and a class of Bohn rule. We find that there is no distinction on simple rules between the exact and false policies if the interest spread is low. However, if the interest spread in the steady state is high, the policy authorities should give up to stabilize inflation and minimize the premium difference. We calculate welfare costs under two policies. if the interest spread is low, there is much difference on welfare costs between two policies. However, if it is high, the difference on the welfare costs are not negligible and is very high. If policy authorities are not aware of the default risk, the policy authorities introduce tighter policy which generates much welfare costs. If the interest spread is high, policy authorities should be aware of the default risk.

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†Corresponding Author. Department of Economics, Mathematics and Statistics, Birkbeck, University of London, Malet Street, London, WC1E 7HX, UK. Phone: +44-20-76316430; E-mail: eiji_okano@econ.nagoya-cu.ac.jp

1 Introduction

Discussion on optimal monetary policy seems matured at a glance. This is also applicable for discussion on optimal monetary and fiscal policy. However, frequent goverment debt crisis gives us discussing optimal monetary policy new research agenda. It is fresh in our memory that the Greek 10-Year credit default swap premium began to soar and reached USD 20,404 on April 2012, the ECB faced increased difficulty in conducting monetary policy. Subsequently, the harmonized consumer price index (HCPI) inflation rate started to increase from -0.6% in July 2009, and the ECB's policy interest rate (the short-run buying operation rate) remained at 1% until April 2011, when HCPI inflation was 2.8%. On the other hand, Greek tax rate was almost constant at that time. These correspondences can be justified from the view point of optimal monetary policy?

Although we refrain answer thsi question immediately, this greek case clearly reveal that there is enough room to discuss optimal monetary and fiscal policy. That is, in this paper, we analyze optimal monetary and fiscal policy amid sovereign debt crisis. Jump to conclusion, we find that policy authorities should adopt monetary and fiscal policy rules which do not severely suppress inflation and do not intend to fiscal retrenchment. In other words, Policy authorities have to stabilize not only inflation but also minimize the premium difference between the (virtual) government debt yield and its coupon rate, which is dubbed simply the premium difference in the text, from the view point of minimizing welfare costs. That is, period welfare costs function which stems from second-order approximated utility function includes not only a quadratic term of inflation but also a quadratic term of the coupon rate gap. This implies that the cost of sovereign risk is summarized the coupon rate gap, analogous to price stickiness generating inflation.

Why does the quadratic term of the premium difference appear on the welfare costs function? Why is the coupon rate gap cost of sovereign risk? At first, we briefly introduce our model to discuss. The model is based on Gali and Monacelli[18] and introduce Uribe's[27] fiscal theory of sovereing risk (FTSR) and is quite simillar to Okano and Inagaki[23], although we assume closed economy different from Gali and Monacelli[18] to simlify. Similar to Gali and Monacelli[18], Calvo pricing is assumed but steady state is distorted because tax is levied on the output similar to Uribe[27]. There are safty assets which si state contingent claims and risky assets which is government debt. This setting corresponds to Uribe[27]. In our model, the FTSR is applicable, similar to Uribe[27]. That is, the government budget constarint is iterated forward and appropriate transversality condition is imposed. The FTSR is based on the fiscal theory of price level (FTPL) which is advocated by Cochrane[10], Leeper[20] and Woodford[29] and the net present value of sum of fiscal surplus dicides not only price level or inflation but also default rate. Suppose a decrease in net present value of fiscal surplus. Under the FTPL, this decreases increases the price level while there is a possibility causing an increase in the default rate, instead of an increase in the price level, under the FTSR.

Now, we discuss our question. If there are just safety assets, households choose consumption schedule in which the intertemporal marginal rate of substition of consumption corresponds to the stocahstic discount factor, namely, inverse of gross nominal interest rate. In our setting, there is both safty assets and risky assets, namley, government debt. If households purchase government debt, households optimal consumption schedule makes the intertemporal marginal rate of substitution corresponding to the inverse of the gross expected retun rate of holding government debt which consists of the government debt yield and the expected default rate. Thus, households has to appropriately adjust their balance of government debt. By adjusting the balance which afects the inverse of the gross expected retun rate of holding government debt through changes in the government debt yield, optimal consumption schedule attains. If the government debt coupon rate precisely corresponds to the government debt yield, such adjustment is not needed. However, that is rare even in actual economy. Thus, this adjustment on the balance of government debt, namely, portfolio rebalance is essential. As shown in the text, the premium gap is function of the expected default rate, apparance of the quadratic term on the period welfare costs funtion implies that the premium gap is the cost of sovereign risk. In other words, sovereign risk generates the cost forcing households to rebalance their portofolio.

We analyze both 'Exact' and 'False'policies. Under the exact policy, exact welfare cost function is minimized by policy authorities, the central bank and the government while the false welfare costs function is minimized by them. The exact welfare cost function is derived exactly and assuming sovereign risk. That is, second order approximated FTSR equation is used to eliminate linear-terms which generate welfare reversal. There are two difference between the exact and the false welfare costs functions, the target level of output and exisetnce of linear quadratic term of the premium gap. The difference in the target level of output between the exact and the false welfare costs functions depends on the interest rate spread in the staeady state, which decides the steady state value of the default rate. If the interest rate spread in the staeady state is zero (the steady ate value of the default rate is zero simultaneously), the target level of output in both welfare costs functions is same. Similarly, existence of the quadratic term of the premium difference on the exact welfare costs function depends on the interest rate spread in the staeady state. If the interest rate spread in the staeady state is zero, the premium difference becomes zero and its quadratic term spontaneously dissapears from the exact welfare costs function. Those two differences depends on the interest rate spread in the staeady state. If the interest rate spread in the staeady state is zero, the exact welfare costs function precisely corresponds to the false welfare costs function. Because the interest rate spread in the staeady state decides the steady state value of the default rate, it can be said that sovereign risk affects period welfare costs function, that is, it affects policy target. If there is sovereign risk, authorities have to pay attention to the target level of output and consider minimizing the premium gap.

This difference on welfare costs function can be ignoired, right? The answer depends on the steady state value of interest rate spread between safty and risky assets. We resort numerical analysis with plausible parameterization and compare the results under the exact policy with the results under the false policy. Further, we also calculate 'optimal' monetary and fiscal policy rules which bring welfare costs close to welfare costs brought by optimal monetary and fiscal policy. Calculating such optimal monetary and fiscal policy rules are attempted by Ferrero[15]. That is, we choose the coefficients on simple rules which are classes of Taylor and Bohn rules to bring welfare costs close to those brought about by optimal monetary and fiscal policy. Interestingly, there are no differences on our Taylor and Bohn rules as long as low steady state value of interest spread which corresponds to roughly average of difference between Long-term government debt yild and the FF rate in the US. However, if the steady state value of interest spread is high and it is corresponds to the value in Greece during Greek debt crisis, The rules are quite different. If the policy authorities do not respect the default risk which generates differences on welfae costs function, they wrongly suppress inflation more and wrongly initiate fical retrenchment.

Finally, to bolster our results from the view point of minimizing welfare costs, we calculate welfare gains from adpting the exact policy. If the steady state value of interest spread is low, the gains are 31% and 0% under the optimal monetary and fiscal policy and simple rules replicating welfare costs brought about by the optimal monetary and fiscal policy, respectively. The gains are not necessarily large or negligible. However, if it is enough high, the gains are 90% and 29% under the optimal monetary and fiscal policy and simple rules. As long as the steady state value of interest spread is enough high, the gains are not negligible. We shall answer 'No' for the previous question. Policy authorities should recognize the default risk and give up to stabilize inflation and fiscal retrenchment.

Due to European debt crisis, there are some influential authors analyzing monetary policy amid sovereing or default risk. Corsetti, Kuester, Meier,and Mueller[12] are one of such authors and argue that fiscal reternchment is essential to stabilize macroeconomy. Their argument is opsite to us. While they do not have viewpoint of minimizing welfare costs function, we have this viewpoint. This difference generates this difference. They focus on suppressing the default. However, the default it self does not necessarily bring welfare costs. Although it is source of the default, the premium difference between the (virtual) government debt yield and the its coupon rate generates welfare costs. Thus, optimal policy is not necessarily fiscal reternchment policy as long as the steady state value of interest spread is enough high.

The remainder of the paper as follows: Section 2 develops the model, section 3 derives the welfare costs function and solves the LQ problem, section 4 is devoted for numerical analysis and section 5 analyzes welfare costs. Appendices provide some technical information.

2 The Model

We introduce firms into Uribe's[27] FTSR and develop a class of DSGE models with nominal rigidities following Gali and Monacelli[18], although we do assume a closed economy.[1] Thus, the default mechanism is quite similar to Uribe[27]. We follow Benigno[4] (an earlier working paper version of Benigno[6]) to clarify the households' choice of risky assets. The household i on the interval $i \in [0,1]$ supplies labor and owns firms. We adopt Calvo pricing and assume that a tax is levied on output and is distorted. Thus, monopolistic power remains, and the steady state is distorted, unlike Gali and Monacelli[18].

Schabert[25] argue that the equilibrium allocation cannot be determined if the central bank sets the interest rate in a conventional way while money supply is controlled, the equilibrium allocation can uniquely be determined under the Uribe's[27] FTSR. We adopt Uribe's[27] FTSR while we do not introduce money into our model. However, this does not definitely involeve that the equilibrium allocation cannot be determined because we follow Benigno[4], as mentioned. Thanks to following Benigno[4], the households' choice of risky assets is determined uniquely thus the equilibrium allocation can uniquely be determined.

2.1 Government

We assume that the total amount of government expenditure is given exogenously in each period by $G_t \equiv \left(\int_0^1 G_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$ where ε denotes the elasticity of substitution among goods. The

[1]Following Ferrero[15], we introduce government into Gali and Monacelli[18]. In other words, the model is a closed economy version of Okano[22].

flow government budget constraint is given by:

$$B_t^n = R_{t-1}^G (1-\delta_t) B_{t-1}^n - \int_0^1 P_t(i)[\tau_t Y_t(i) - G_t(i)] \, di.$$

where $R_t^G \equiv R_t \Gamma(-sp_t)$ denotes the government debt coupon rate, $R_t \equiv 1 + r_t$ denotes the gross (risk-free) nominal interest rate, r_t is the net interest rate, B_t^n is the nominal government debt, δ_t is the default rate, $sp_t \equiv \frac{SP_t}{SP} - 1$ is the percentage deviation of the (real) fiscal surplus from its steady-state value, $SP_t \equiv \tau_t Y_t - G_t$ denotes the (real) fiscal surplus, τ_t denotes the tax rate. Notice that we define $Y_t \equiv \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$ where Y_t denotes (aggregated) output. Because the government expenditure is given exogenously, fiscal policy consists of choosing the mix between taxes and the one-period nominal debt with sovereign risk to finance the exogenous process of government expenditure.

Here, we discuss the government debt coupon rate $R_t^G \equiv R_t \Gamma(-sp_t)$, where $\Gamma'(-sp_t) > 0$ by assumption. Our assumption implies that government decides the government debt coupon rate depending on its fiscal situation, such that if the fiscal situation worsens, the government increases the coupon rate. Note that the government debt coupon rate R_t^G is not government debt yield, which is fully endogenized. In our setting, the government debt yield is decided by households' intertemporal optimal condition; namely, the Euler equation. Thus, the government debt yield is decided endogenously, although the government debt coupon rate depends on our assumption.

As mentioned, the function $\Gamma(-sp_t)$ is hinted at by Benigno[4], who develops a two-country model with imperfect financial integration, although the details are somewhat different from Benigno[4]. Benigno[4] assumes that households in the home country face a burden in international financial markets. As borrowers, households in the home country will be charged a premium on the foreign interest rate; as lenders, they will receive remuneration less than the foreign interest rate. Following his setting, Benigno[4] assumes $\Gamma'(\cdot) < 0$, which implies that the higher the foreign country's government debt, the lower the remuneration for holding the foreign country's government debt.[2] However, on the contrary, our setting implies that the lower the fiscal surplus, the less the remuneration for holding government debt owing to default, which in turn harms capital and makes households hesitate to hold government debt. The government has to pay additional remuneration for holding government debt, which provides households with a motivation for doing so. Thus, we assume that $\Gamma'(\cdot) > 0$. That is, the lower the fiscal surplus, the higher the interest rate multiplier.

Another assumption that differs from Benigno[4] is that $\Gamma(\cdot)$ is a function of the fiscal surplus, while Benigno[4] assumes that it is a function of current government debt with an interest payment; that is, $R_t B_t$. Our setting for $\Gamma(\cdot)$ follows Corsetti, Kuester, Meier and Mueller[12] indirectly. Corsetti, Kuester, Meier and Mueller[12] assume that the higher the fiscal deficit, the greater the probability of default, and vice versa. If we are given that the higher the probability of default, the higher the government debt coupon rate, our assumption that $\Gamma(\cdot)$ is a decreasing function of the fiscal surplus is consistent with their analysis because the assumption implies that the higher the fiscal surplus, the higher the government debt coupon rate. That is, if we are given that the higher the probability of default, the higher the government debt coupon rate, it can be said that we indirectly assume that the lower the fiscal surplus, the higher the default rate, and this is

[2] Benigno[4] observes that this function, which depends only on the level of real government bonds in his setting, captures the costs of undertaking positions in the international asset market or the existence of intermediaries in the foreign asset market.

similar to Corsetti, Kuester, Meier and Mueller[12]. Furthermore, our setting on $\Gamma(\cdot)$ is supported by some empirical evidence. We analyze whether a fiscal deficit or government debt with interest payment increases the interest rate multiplier $\Gamma(\cdot)$ using Greek data. These data imply that the fiscal deficit but not government debt with interest payment increases $\Gamma(\cdot)$.[3] Thus, our assumption regarding $\Gamma(\cdot)$ is consistent with some previous work and the available data.

The log-linearized definition of the fiscal surplus is given by:

$$sp_t = \varsigma_\tau \hat{\tau}_t + \varsigma_\tau y_t - \frac{\varsigma_\tau \sigma_G}{\tau} g_t - \frac{\varsigma_\tau \sigma_G}{\tau} \hat{\zeta}_t. \tag{1}$$

where $\varsigma_\tau \equiv \frac{\tau}{\frac{SP}{Y}}$ denotes the tax revenue elasticity and $\sigma_G \equiv \frac{G}{Y}$ denotes the steady-state share of the government expenditure to output, $\hat{\tau}_t \equiv \frac{d\tau_t}{\tau}$ denotes the percentage deviation of the tax rate from its steady-state value. We simply refer to the percentage deviation of the tax rate from its steady-state value $\hat{\tau}_t$ as the tax gap.

By solving cost-minimization problems, the optimal allocation of generic goods is given by $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t$ and $G_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} G_t$, the previous flow government budget constraint can be rewritten as:

$$B_t^n = R_{t-1}^G (1 - \delta_t) B_{t-1}^n - P_t SP_t,$$

where:

$$P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} dh\right]^{\frac{1}{1-\varepsilon}}, \tag{2}$$

denotes the price level. Dividing both sides of the equality by P_t yields:

$$B_t = R_{t-1}^G (1 - \delta_t) B_{t-1} \Pi_t^{-1} - SP_t. \tag{3}$$

with $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ being the gross inflation rate. The first term on the RHS corresponds to the amount of redemption with the nominal interest payment and shows that the lower the past fiscal surplus, the higher the interest payments, and the higher the default rate, the lower the redemption, and vice versa.

Log-linearizing Eq.(3) yields:

$$b_t = \frac{\tau}{\beta(\tau + \phi\varsigma_\tau\sigma_B)} \hat{r}_{t-1}^G - \frac{\phi\varsigma_\tau\sigma_B}{\beta(\tau + \phi\varsigma_\tau\sigma_B)} \hat{\delta}_t + \frac{\tau}{\beta(\tau + \phi\varsigma_\tau\sigma_B)} b_{t-1} - \frac{\tau}{\beta(\tau + \phi\varsigma_\tau\sigma_B)} \pi_t - \frac{\tau}{\varsigma_\tau\sigma_B} sp_t. \tag{4}$$

Here, we show that the log-linearized definition of the government debt coupon rate is given by:

$$\hat{r}_t^G = \hat{r}_t - \phi sp_t \tag{5}$$

with $\hat{r}_t^G \equiv \frac{dR_t^G}{R^G}$.

2.2 Households

2.2.1 The FONCs for Households

A representative household's preference is given by:

$$\mathcal{U} \equiv \mathrm{E}_0 \left(\sum_{t=0}^{\infty} \beta^t U_t\right), \tag{6}$$

[3] See Appendix C for details.

where $U_t \equiv \ln C_t - \frac{1}{1+\psi} N_t^{1+\psi}$ denotes the period utility, E_t is the expectation conditional on the information set at period t, $\beta \in (0,1)$ is the subjective discount factor, C_t is the consumption index, $N_t \equiv \int_0^1 N_t(i)\, dh$ is the hours of labor, and ψ is the inverse of the elasticity of labor supply.

The consumption index of the continuum of differentiated goods is defined as follows:

$$C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{7}$$

where $\varepsilon > 1$ is the elasticity of substitution across goods.

The maximization of Eq.(6) is subject to a sequence of intertemporal budget constraint of the form:

$$R_{t-1} D_{t-1}^n + R_{t-1}^G B_{t-1}^n (1-\delta_t) + W_t N_t + PR_t \geq \int_0^1 P_t(i)\, C_t(i)\, di + D_t^n + B_t^n, \tag{8}$$

where D_t^n denotes the nominal state contingent claim, W_t is the nominal wage, PR_t denotes profits from the ownership of the firms, Furthermore, we define V as the steady-state value of any variables V_t and v_t as the percentage deviation of V_t from its steady-state value. Thus, SP is the steady-state value of the fiscal surplus.

By solving cost-minimization problems for households, we have the optimal allocation of expenditures as follows:

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t. \tag{9}$$

Once we account for Eq.(9), the intertemporal budget constraint can be rewritten as:

$$R_{t-1} D_{t-1}^n + R_{t-1}^G B_{t-1}^n (1-\delta_t) + W_t N_t + PR_t \geq P_t C_t + D_t^n + B_t^n.$$

The remaining optimality conditions for the household's problem are given by:

$$\beta \mathrm{E}_t \left(\frac{P_t C_t}{P_{t+1} C_{t+1}} \right) = \frac{1}{R_t}, \tag{10}$$

which is the intertemporal optimality condition—namely, the Euler equation—and:

$$C_t N_t^{\psi} = \frac{W_t}{P_t}, \tag{11}$$

which is the standard intratemporal optimality condition.

There is another intertemporal optimality condition depicting the households' motivation to hold government debt with default risk. This is obtained by differentiating the Lagrangian by government nominal debt and is given by:

$$\beta \mathrm{E}_t \left(\frac{P_t C_t}{P_{t+1} C_{t+1}} \right) = \frac{1}{R_t^H \mathrm{E}_t (1-\delta_{t+1})}. \tag{12}$$

with $R_t^H \equiv R_t \left\{ \Gamma(-sp_t) + B_t \Gamma'(-sp_t)\, SP^{-1} \right\}$, and R_t^H can be interpreted as the (gross) government debt yield (excluding the default risk). The definition of government debt yield R_t^H implies that households adjust its portforio of government debt to make government debt yield considering default risk $R_t^H \mathrm{E}_t (1-\delta_{t+1})$ correspond to the inverse of marginal rate of consumption, namely, R_t, as long as additional interest payment for holding government debt is not sufficient to realize optimal consumption schedule by holding government debt.

In fact, Combining Eqs.(10) and (12), we have:

$$R_t = R_t^H \mathrm{E}_t \left(1 - \delta_{t+1}\right), \tag{13}$$

which shows that the marginal rate of substitution for consumption is the same for households holding either (real) state-contingent claims D_t or (real) government debt B_t because both R_t and $R_t^H \mathrm{E}_t \left(1 - \delta_{t+1}\right)$ equal the marginal rate of substitution $\beta \mathrm{E}_t \left(\frac{P_t C_t}{P_{t+1} C_{t+1}}\right)$. That is, the consumption schedule is the same whether households hold state-contingent claims D_t or government debt B_t.

Rearrangig Eq.(13) yields:

$$R_t^S = \mathrm{E}_t \left(1 - \delta_{t+1}\right)^{-1}$$

where $R_t^S \equiv \frac{R_t^H}{R_t}$ denotes the (gross) interest rate spread for holding the government debt, namely, risky asstes. This equality shows that the higer the expected default rate, the higher the interest rate spread and vice verca.

Log-linearizing Eq.(13) yields:

$$\hat{r}_t^S = \frac{\phi \varsigma_\tau \sigma_B}{\tau} \mathrm{E}_t \left(\delta_{t+1}\right), \tag{14}$$

with $\hat{r}_t^S \equiv \frac{dR_t^S}{R^S}$ where $\hat{r}_t \equiv \frac{dR_t}{R}$ and $\hat{r}_t^H \equiv \frac{dR_t^H}{R^H}$ denotes the nominal interest gap and the government debt yield gap, respectively, and $\sigma_B \equiv \frac{B}{Y}$ denotes the steady state share of the the government debt to the output.

Log-linearizing the definition of government debt yield R_t^H, we have:

$$\hat{r}_t^S = -\frac{\phi \left(\tau + \gamma \varsigma_\tau \sigma_B\right)}{\tau + \phi \varsigma_\tau \sigma_B} sp_t + \frac{\phi \varsigma_\tau \sigma_B}{\tau + \phi \varsigma_\tau \sigma_B} b_t, \tag{15}$$

where $\phi \equiv \Gamma'(0)$ denotes the interest rate spread in the steady state and $\gamma \equiv \frac{\Gamma''(0)}{\Gamma'(0)}$ denotes the elasticity of the interest rate spread to a one percent change in the fiscal deficit in the steady state. Following Benigno[4], we define the interest rate spread for government debtϕ and assume $\Gamma(0) = 1$. The elasticity γ is an unfamiliar parameter, and we assume $\mid \Gamma'(\cdot) \mid < \mid \Gamma''(\cdot) \mid$; thus, $\gamma > 1$. Our assumption implies that a decrease in the fiscal surplus increases the government debt coupon rate via an increase in the interest rate multiplier, and vice versa, and that changes in the government debt coupon rate are larger than the changes in the fiscal surplus in absolute value.[4]

Given our assumption, Eq.(15) implies that an increase in the fiscal surplus decreases the government debt yield, and vice versa. This is intuitively consistent because an increase in fiscal surplus decreases the interest rate multiplier and decreases the government debt yield. In addition, in the third term on the right-hand side (RHS), the sign is positive. This shows that the government debt yield is an increasing function of government debt. An increase in government debt coincides with a decrease in the fiscal surplus, and vice versa. Thus, this positive sign is consistent with the negative sign in the second term. That is, an increase in government debt increases the government debt yield through an increase in the interest rate multiplier $\Gamma(\cdot)$, which is brought about by a decrease in the fiscal surplus.

2.2.2 Fiscal Theory of Sovereign Risk

The appropriate transversality condition for government debt is given by:

$$\lim_{j \to \infty} \beta^{t+j+1} \mathrm{E}_t \left[R_{t+j}^G \left(1 - \delta_{t+j+1}\right) \frac{P_{t+j} B_{t+j}}{P_{t+j+1}}\right] = 0.$$

[4]Our assumption $\gamma > 1$ is supported by the data. See Okano and Inagaki[23] for details.

By iterating the second equality in Eq.(3) forward, plugging Eq.(10) into this iterated equality, and imposing the appropriate transversality condition for government debt, we have:

$$C_t^{-1} R_{t-1}^G B_{t-1} \Pi_t^{-1} (1 - \delta_t) = C_t^{-1} SP_t + \beta \frac{R_t^H}{R_t^G} \mathrm{E}_t \left(C_{t+1}^{-1} SP_{t+1} \right) + \beta^2 \mathrm{E}_t \left(\frac{R_t^H}{R_t^G} \frac{R_{t+1}^H}{R_{t+1}^G} C_{t+2}^{-1} SP_{t+2} \right) + \cdots, \quad (16)$$

which roughly shows that the burden of government debt redemption with interest payment in terms of consumption, or the left-hand side (LHS), corresponds to the expected sum of the discounted value of the fiscal surplus in terms of consumption, or the RHS, because of the transversality condition. Here, $\frac{R_t^H}{R_t^G}$ and so forth appear on the RHS. An increase in the government debt coupon rate R_t^G then worsens the fiscal situation through the increase in the interest payment. Thus, R_t^G is the denominator. An increase in the government debt yield facilitates the purchase of government debt even though consumption decreases. A decrease in the consumption then improves the fiscal situation because the decrease in the consumption increases the fiscal surplus in terms of consumption. Thus, R_t^H appears as the numerator.

Eq.(16) can be rewritten as:

$$\delta_t = 1 - \frac{\left(R_{t-1}^R \right)^{-1} \sum_{k=0}^{\infty} \prod_{h=0}^{k} \beta^k \mathrm{E}_t \left(R_{t+h-1}^R C_{t+k}^{-1} SP_{t+k} \right)}{C_t^{-1} R_{t-1}^G B_{t-1} \Pi_t^{-1}}, \quad (17)$$

where $R_t^R \equiv \frac{R_t^H}{R_t^G}$ denotes the (gross) premium difference between the government debt yield and its coupon rate. Eq.(17) is our FTSR and implies that an increase in inflation does not necessarily occur even if the government's solvency is lost, and vice versa, similar to Uribe[27]. Not only inflation, but also default, can mitigate the burden of government debt redemption with interest payment. Suppose that the price level is constant and there is no inflation. In this situation, if the net present value of the fiscal surplus in terms of consumption (the numerator) is about to fall below the burden of government debt redemption with interest payment in terms of consumption (the denominator), the second term on the RHS is less than unity. Simultaneously, the LHS exceeds zero; that is, default occurs. In other words, if the government falls insolvent while the price level is strictly stable, default is inevitable. Uribe[27] shows the SI–SD trade-off by introducing default—namely, sovereign risk—into the central equation of the fiscal theory of the price level (FTPL). Similar to Uribe[27], at first glance, Eq.(17) also implies that there is an SI–SD trade-off. Furthermore, he calibrates his model and compares the Taylor rule that stabilizes inflation with the interest rate peg. Under the interest rate peg, the interest rate on risky assets corresponds to the risk-free asset interest rate pegged to the steady-state rate. This calibration shows that default ceases just one period after the shock decreasing the fiscal surplus, even though default continues under the Taylor rule after the shock. This implies that a Taylor rule to stabilize inflation includes the unwelcome possibility of magnifying sovereign risk, and this calls for an interest rate peg to counter default. Although Uribe[27] ignores the welfare perspective of these actions, his policy implications are persuasive. Paying attention to just Eq.(17), which is similar to that in Uribe's[27] model, we seem to obtain policy implications quite similar to those in Uribe[27].

We now present the relationship between our FTSR; namely, Eq.(17) and the FTPL. If there is neither default risk nor an interest rate multiplier in Eq.(17), Eq.(17) reduces to the following because of $R_t^G = R_t^H = R_t$:

$$1 = \frac{\sum_{k=0}^{\infty} \beta^k \mathrm{E}_t \left(C_{t+k}^{-1} SP_{t+k} \right)}{C_t^{-1} R_{t-1} B_{t-1} \Pi_t^{-1}}, \quad (18)$$

which is our version of the FTPL. On the RHS in this equality, the numerator is the net present value of the sum of the fiscal surplus in terms of consumption, and the denominator is the burden of the government debt redemption with interest payment in terms of consumption divided by inflation. The LHS is unity. If solvency worsens, the price level increases; that is, inflation occurs, such that the burden of government debt redemption is mitigated. For now, we introduce sovereign risk, and this mechanism is no longer fully applicable, as Eq.(17) implies.

2.2.3 Relationship between Default Rate and Fiscal Surplus

By leading Eq.(17) one period and plugging this into Eq.(17) itself, we can rewrite Eq.(17) as a second-order differential equation as follows:

$$\delta_t = 1 - \frac{1}{R_{t-1}^G \Pi_t^{-1} B_{t-1}} \left\{ SP_t + \beta \mathrm{E}_t \left[\left(\frac{C_t}{C_{t+1}} \Pi_{t+1}^{-1} \right) R_t^H \left(1 - \delta_{t+1}\right) B_t \right] \right\}. \tag{19}$$

In Eq.(19), the current government debt B_t appears in the second term on the RHS and the sign is negative. That is, a decrease in current government debt increases the default rate, and vice versa. Why is the sign of government debt B_t in the second term on the RHS negative? This stems from the transversality condition for government debt. Because of the transversality condition, Eq.(17) and its second-order differential version Eq.(19) are strictly applicable. That is, once issued, government debt must be redeemed. Otherwise, the burden of redemption is mitigated by default or inflation. To keep Eq.(17), once government debt is issued, the fiscal surplus must be improved while newly issued government debt is about to reduce the fiscal surplus. Because the fiscal surplus must improve to redeem debt, the default rate declines as a result of an improvement in the fiscal surplus when government debt increases. Thus, the sign is negative.

In addition, we can easily imagine that the fiscal surplus is a function of the output gap. By using Gali and Monacelli's[18] definition of the output gap—namely, $\tilde{y}_t \equiv y_t - \bar{y}_t$, where $\tilde{y}_t$ and $\bar{y}_t$ denote the output gap and the natural rate of output, respectively—we can recognize that stabilizing the fiscal surplus leads to the stabilization of the output gap.[5] As mentioned, we need to stabilize the default rate to stabilize the fiscal surplus. In addition, stabilizing the fiscal surplus leads to the stabilization of the output gap, which positively links with inflation under Calvo pricing. Thus, we can imagine that there is not necessarily an SI–SD trade-off.

Log-linearizing Eq.(19) yields:

$$\begin{aligned} c_t \;=\; & \mathrm{E}_t\left(c_{t+1}\right) - \hat{r}_t^H + \frac{\phi \varsigma_\tau \sigma_B}{\tau} \mathrm{E}_t\left(\hat{\delta}_{t+1}\right) + \mathrm{E}_t\left(\pi_{t+1}\right) - b_t + \frac{\tau + \varsigma_\tau \sigma_B}{\varsigma_\tau \sigma_B} \hat{r}_{t-1}^G - \frac{\phi \varsigma_\tau \sigma_B}{\beta\left(\tau + \phi \varsigma_\tau \sigma_B\right)} \hat{\delta}_t \\ & - \frac{\tau + \varsigma_\tau \sigma_B}{\varsigma_\tau \sigma_B} \pi_t + \frac{\tau + \varsigma_\tau \sigma_B}{\varsigma_\tau \sigma_B} b_{t-1} - \frac{\tau}{\varsigma_\tau \sigma_B} sp_t, \end{aligned} \tag{20}$$

which is our log-linearized Euler equation.

2.3 Firms

This subsection outlines the production, price setting, marginal cost, and features of the firms, and these are quite similar to Gali and Monacelli[18], although here the tax is levied on firm sales and is not constant.[6]

[5] In our model, the steady state is not efficient because friction stemming from the monopolistically competitive market cannot be dissolved by taxation. Thus, the target level of the output gap (or efficient output gap) is not zero, even though the target level is zero in Gali and Monacelli[18], because the steady state is efficient.

[6] Unlike our setting, Gali and Monacelli[18] assume that under constant employment subsidies, monopolistic power completely disappears.

A typical firm in each country produces a differentiated good with a linear technology represented by the production function:

$$Y_t(i) = A_t N_t(i),$$

where A_t denotes the productivity.

By combining the production function and the optimal allocation for goods $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t$, we have an aggregate production function relating to aggregate employment as follows:

$$N_t = \frac{Y_t Z_t}{A_t}, \tag{21}$$

where $Z_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di$ denotes the price dispersion.

Log-linearizing Eq.(21) yields:

$$n_t = y_t - a_t. \tag{22}$$

We assume that productivity follows an AR(1) process; namely, $\mathrm{E}_t(a_{t+1}) = \rho_A a_t$, similar to government expenditure. Z_t disappears in Eq.(17) because of $o\left(\|\xi\|^2\right)$.

Each firm is a monopolistic producer of one of the differentiated goods. Each firm sets its prices $P_t(i)$ taking as given P_t and C_t. We assume that firms set prices in a staggered fashion, Calvo pricing, according to which each seller has the opportunity to change its price with a given probability $1-\theta$, where an individual firm's probability of reoptimizing in any given period is independent of the time elapsed since it last reset its price. When a firm has the opportunity to set a new price in period t, it does so in order to maximize the expected discounted value of its net profits. The FONCs for firms are given by:

$$\tilde{P}_t = \frac{\mathrm{E}_t\left(\sum_{k=0}^{\infty} \theta^k \beta^k \tilde{Y}_{t+k} \frac{\varepsilon}{\varepsilon-1} P_{t+k} MC_{t+k}\right)}{\mathrm{E}_t\left(\sum_{k=0}^{\infty} \theta^k \beta^k \tilde{Y}_{t+k}\right)}, \tag{23}$$

where $MC_t \equiv \frac{W_t}{(1-\tau_t)P_t A_t}$ denotes the real marginal cost, $\tilde{Y}_{t+k} \equiv \left(\frac{\tilde{P}_t}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}$ denotes the demand for goods when firms choose a new price, and $\tilde{P}_t$ denotes the newly set prices. Note that we assume that government levies a tax on firm sales.

By log-linearizing Eq.(23), we have:

$$\pi_t = \beta \mathrm{E}_t(\pi_{t+1}) + \kappa mc_t, \tag{24}$$

with $\kappa \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$ being the slope of the New Keynesian Phillips Curve (NKPC). Eq.(24) is the fundamental equality of our NKPC.

Substituting Eq.(11) into the definition of the real marginal cost yields:

$$MC_t = \frac{C_t N_t^{\psi}}{(1-\tau_t) A_t}. \tag{25}$$

Note that the marginal cost in the steady state, which is the inverse of a constant markup, is smaller than one, while the gross tax rate $1-\tau$ is definitely smaller than one. In such a case, the steady-state wedge between the marginal product of labor and the marginal utility of consumption is not unity. That is, monopolistic power remains because it is unable to be completely absorbed through taxation. As we discuss later, we need to derive our welfare criteria following Benigno and Woodford[8] because monopolistic power is no longer removed completely, and the steady state is distorted.

Log-linearizing Eq.(25) yields:

$$mc_t = c_t + \psi n_t + \frac{\tau}{1-\tau}\hat{\tau}_t - a_t. \tag{26}$$

2.4 Equilibrium

The market-clearing condition requires:

$$Y_t(i) = C_t(i) + G_t(i),$$

for all $i \in [0,1]$ and all t. By plugging the optimal allocation for generic goods including Eq.(8) into this market-clearing condition, we have:

$$Y_t = C_t + G_t. \tag{27}$$

By log-linearizing Eq.(27), we obtain:

$$y_t = \sigma_C c_t + \sigma_G g_t, \tag{28}$$

where $\sigma_C \equiv 1 - \sigma_G$ denotes the steady-state ratio of consumption to output.

3 Welfare Costs and the LQ Problem

3.1 Derivation of Welfare Costs Function

Following Gali[16], the second-order approximated utility function is given by:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{U_t - U}{U_C C}\right) &= \sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{\Phi}{\sigma_C}y_t\right) - \sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left[\frac{(1-\Phi)(1+\psi)}{\sigma_C 2}y_t^2 - \frac{(1-\Phi)(1+\psi)}{\sigma_C}y_t a_t\right.\\ &\quad\left. + \frac{\Lambda_\pi}{2}\pi_t^2\right] + \text{t.i.p.} + o\left(\|\xi\|^3\right),\end{aligned} \tag{29}$$

with $\Lambda_\pi \equiv \frac{(1-\Phi)\varepsilon}{\sigma_C \kappa}$ where t.i.p. denotes the terms independent of policy, $o\left(\|\xi\|^3\right)$ are the terms of order three or higher, and $\Phi \equiv 1 - \frac{1-\tau}{\frac{\varepsilon}{\varepsilon-1}}$ denotes the steady-state wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor. On the RHS, there is a linear term $\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{\Phi}{\sigma_C}y_t\right)$ generating the welfare reversal which must be eliminated to avoid welfare revarsal. By using second-order approximated AS equation Eq.(23), second-order approximated market clearing condition Eq.(27) and second-order approximated government solvency condition Eq.(17), we have:

$$\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{\Phi}{\sigma_C}y_t\right) = -\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left[\tilde{\Omega}_1 y_t^2 - 2y_t\left(\Omega_2 g_t + \tilde{\Omega}_3 a_t\right) + \frac{\Lambda_r}{2}\left(\hat{r}_t^R\right)^2\right] + \Upsilon_0 + \text{t.i.p.} + o\left(\|\xi\|^3\right),$$

where $\hat{r}_t^R \equiv \frac{dR_t^R}{R^R}$ denotes the premium difference, $\Upsilon_0 \equiv \Theta_1(1-\beta-\delta)^{-1}\hat{\mathcal{W}}_0 + \Theta_2\kappa^{-1}\mathcal{V}_0$ denotes transitory component, $\mathcal{V}_0$ denotes the initial value of second-order approximated AS equation, $\mathcal{W}_0$ denotes the initial value of second-order approximated government solvency condition and $\tilde{\Omega}_1$, Ω_2,

$\tilde{\Omega}_3$, Ω_4 are complicated building block of parameters with $\hat{\mathcal{W}}_t \equiv \frac{\mathcal{W}_t - \mathcal{W}_{t-1}}{\mathcal{W}_{t-1}}$, $\Lambda_r \equiv \frac{\Theta_1 \beta\left[1+(\gamma-1)^2\right]}{(1-\beta-\delta)(\gamma-1)^2}$, $\Theta_1 \equiv \frac{(1-\beta-\delta)\tau\Phi}{\Xi_0}$, $\Theta_2 \equiv -\frac{\Phi(1+2\omega_g)\omega_\phi(1-\tau)}{\Xi_0}$, $\omega_\phi \equiv 1-\beta\left(1-\phi\right) - \frac{\phi\varsigma_\tau \sigma_B}{\tau+\phi\varsigma_\tau\sigma_B}$ and Ξ_0 being complicated building of parameters.

Plugging the previous expression into Eq.(29) yields:

$$\sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left(\frac{U_t - U}{U_C C}\right) = \sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left(L_t\right) + \Upsilon_0 + \text{t.i.p.} + o\left(\|\xi\|^3\right),$$

which is a second-order approximated utility function and the linear term is appropriately eliminated, where:

$$L_t \equiv \frac{\Lambda_y}{2}\left(y_t - y_t^*\right)^2 + \frac{\Lambda_\pi}{2}\pi_t^2 + \frac{\Lambda_r}{2}\left(\hat{r}_t^R\right)^2 \tag{30}$$

denotes the period welfare costs function, $y_t^* \equiv \frac{\Omega_2}{\Omega_1} g_t + \frac{\Omega_3}{\Omega_1} a_t$, denotes the efficient level of output with $\Lambda_y \equiv 2\Omega_1$, $\Omega_1 \equiv \tilde{\Omega}_1 + \frac{(1+\psi)(1-\Phi)}{2\sigma_C}$.

Period welfare costs function Eq.(30) has distinctive features. To clarify distinctive features, we derive the period welfare costs function by using second-order approximated government solvency condition which is derived from Eq.(18). Remember that Eq.(18) is our version of the FTPL and can be derived from the government solvency condition Eq.(17) by assuming that there is neither default risk nor an interest rate multiplyer. Now, the period welfare costs function by using second-order approximated government solvency condition which is derived from Eq.(18) is given by:

$$L_t^f = \frac{\Lambda_y^f}{2}\left(y_t - y_t^f\right)^2 + \frac{\Lambda_\pi}{2}\pi_t^2. \tag{31}$$

which is analogous to the period welfare costs function derived by Benigno and Woodford[9] where $y_t^f \equiv \frac{\Omega_2^f}{\Omega_1^f} g_t + \frac{\Omega_3^f}{\Omega_1^f} a_t$ denotes the efficient level of output when there is no default risk with $\Lambda_y^f \equiv 2\Omega_1^f$, $\Omega_1^f \equiv \tilde{\Omega}_1^f + \frac{(1+\psi)(1-\Phi)}{2\sigma_C}$, $\Omega_3^f \equiv \tilde{\Omega}_3^f + \frac{(1+\psi)(1-\Phi)}{2\sigma_C}$ and Ω_2^f being complicated block of parameters. There is not the quadratic term of $\hat{r}_t^R$, Λf_y replaces Λ_y and target level of output is not y_t^* but y_t^f in Eq.(30).

3.1.1 Welfare Costs in an Economy with Sovereign Risk

The most notable feature of welfare costs in an economy with sovereign risk, namely Eq.(30), is the quadratic term of the premium difference between the government debt yield and the government debt coupon rate $\hat{r}_t^R$ which is the third term in the RHS. Apparance of the this term suggests that there is an opportunity costs on holding government debt. When households hold the government debt, households can obtain inerest income which is not necessarilly at the government debt coupon rate r_t^G but may be at the government debt yield r_t^H. To choose optimal consumption schedule Eq.(12), households has to maneuver their government debt possition B_t sufficing the definition of the (gross) government debt yield as follows:

$$\begin{aligned} R_t^H &= R_t\left\{\Gamma\left(-sp_t\right) + B_t \Gamma'\left(-sp_t\right) SP^{-1}\right\} \\ &= R_t^G + R_t B_t \Gamma'\left(-sp_t\right) SP^{-1}. \end{aligned}$$

As shown in the previous exression, the government debt coupon rate does not necessarily corresponds to the government debt yield and households has to abandon income at the government debt coupon rate but has to obtain income at the government debt yield on their own government debt. Thus, there is an opportunity cost on maneuvering the government debt possition. If they neglect to maneuver, households can no longer choose optimal consumption schedule and there results welfare costs. This the reason why there is a quadratic of the premium difference between the government debt yield and the government debt coupon rate $\hat{r}_t^R$ on Eq.(30).

Apparance of The quadratic of the premium difference between the government debt yield and the government debt coupon rate in Eq.(30) depends on sovereign risk. The third term on the RHS in Eq.(30) can be rewritten as:

$$\Lambda_r \left(r_t^R\right)^2 = \Lambda_\delta \mathrm{E}_t \left(\hat{\delta}_{t+1} - \hat{\delta}_{t+1}^*\right)^2,$$

because of:

$$\hat{r}_t^R = \frac{\phi \varsigma_\tau \sigma_B}{\tau} \mathrm{E}_t \left(\hat{\delta}_{t+1} - \hat{\delta}_{t+1}^*\right), \tag{32}$$

which is derived by using Eqs.(5) and (14) with $\Lambda_\delta \equiv \Lambda_r \phi^2 \left(\frac{\varsigma_\tau \sigma_B}{\tau}\right)^2$ where $\hat{\delta}_t^* \equiv -\frac{\tau}{\varsigma_\tau \varsigma_B} sp_{t-1}$ denotes the target level of default rate. The previous expression shows the welfare costs stemming from the third term on Eq.(30) is welfare costs of the deviation of the expected default gap from its target level $\mathrm{E}_t \left(\hat{\delta}_{t+1}^*\right)$ corresponding to (percentage deviation of) fiscal deficit from its steady state value $-sp_t$. In addition, the previous expresshon shows that the higher the steady state value of the interest spread ϕ, the higher the weights on the deviation of the expected default gap from its target level Λ_δ. Then, we have to pay attention to the steady state value of interest spread ϕ which detremins the steady state value of the default rate because of:

$$\delta = \frac{\phi \varsigma_\tau \sigma_B}{\tau + \phi \varsigma_\tau \sigma_B}.$$

That is, the higher the the steady state value of interest spread, the higher the steady state value of the default, and vice versa. Because of that the higher the steady state value of the interest spread ϕ, the higher the weights on the deviation of the expected default gap from its target level Λ_δ, the higer the staedy state value of the default rate δ, the higher the weights on the deviation of the expected default gap from its target level Λ_δ. In addition, when there is no interets spread in the steady state which causes that the stady state value of the default rate is zero, $\Lambda_\delta = 0$ is applied and the third term on Eq.(30) $\frac{\Lambda_r}{2} \left(\hat{r}_t^R\right)^2$ dissappers.

Another distinctive features on Eq.(30) against to Eq.(31) are that the weights on the output deviation from its target level Λ_y replaces Λ_f and the target level output y_t^* replaces y_t^f. While Λ_y^f in Eq.(31) does not depend on the steady state value of the interest spread ϕ, Λ_y depends on ϕ. When $\phi = 0$, $\Lambda_y = \Lambda_y^f$ is applied. That is, difference on the weights on the output deviation from its target level depends on the steady state value of the interest spread. In Eq.(30), the target level output y_t^* replaces y_t^f and y_t^* depends on ϕ although y_t^f does not depend on it. When $\phi = 0$, $y_t^* = y_t^f$.

Fig.1 shows feature of welfare costs function focusing on various value of the steady state value of the interest spread ϕ. Panel 1 in Fig.1 shows that the higher the steady state value of the interest spread or the sovereign risk, the higher the weights for deviation of the expected default rate from its target $\Lambda_\delta = \Lambda_r \phi^2$, and vice varsa. Panel 2 in Fig.1 shows that the higher the steady state value of the interest spread, the lower the weights on welfare relevant output gap Λ_y and vice varsa. Panel 3 in Fig.1 calculate difference on the targetlevel output $y_t^* - y_t^f = \frac{\Omega_1^f \Omega_2 - \Omega_1 \Omega_2^f}{\Omega_1 \Omega_1^f} g_t + \frac{\Omega_1^f \Omega_3 - \Omega_1 \Omega_3^f}{\Omega_1 \Omega_1^f} a_t$

with one percent change in both the productivity and the government expenditure and shows that the higher the steady state value of the interest spread, the higher the difference on the targetlevel output. This implies that the sovereign risk magnifies monopolistical competitive power which cannot be dissolved by distorted taxation. Based on those things, when $\phi = 0$ in which the steady state value of the default rate is zero, namely $\delta = 0$, Eq.(31) boils down to Eq.(30), that is:

$$L_t^* = L_t^f$$

is applied. If there is not sovereign risk, welfare costs function is analogous to one derived by Benigno and Woodford[9] who do not assume sovereign risk. Thus, it can be said that sovereign risk changes the form of welfare costs function.

3.2 The LQ Problem

The policy authorities minimize Eq.(30) or Eq.(31) for all t subject to Eqs.(1), (4), (5), (14), (15), (20), (22), (24), (26) and (28) and select the sequence $\left\{y_t, \pi_t, \hat{r}_t^G, \hat{r}_t, \hat{r}_t^H, c_t, b_t, mc_t, n_t, sp_t, \hat{\tau}_t, \hat{\delta}_t\right\}_{t=0}^{\infty}$. We designate the policy minimizing Eq.(30) 'Exact' policy because there is sovereign risk and policy authorities recognize sovereign risk while we designate the policy minimizing Eq.(31) 'False' policy because there is sovereign risk but policy authorities do not recognize sovereign risk. Eq.(30) and Eq.(31) are not only distinguished by the quadratic term of the premium difference $\hat{r}_t^R$ but also the weights on the output deviation from its target level and the target level output. Comparing outcome of policy minimizing Eq.(30) with minimizing Eq.(30) without the quadratic term of the premium difference cannot analyze how sovereign risk affects outcome of optimal policy. We have to consider not only the quadratic term of the premium difference but also the weights on the output deviation from its target level and the target level output. Thus, we compare 'Exact' policy which minimizes Eq.(30) with 'False' policy which minimizes Eq.(31).

Under the Exact policy, the policy authorities minimize Eq.(30) while the policy authorities minimize Eq.(31) under the false policy. In the following, we introduce some FONCs which are worth to discuss.

The FONCs for the output are given by:

$$\Lambda_y y_t = \varsigma_\tau \rho_{7,t} + \rho_{8,t} - \rho_{10,t} + \Lambda_y y_t^*, \quad (33)$$

$$\Lambda_y^f y_t = \varsigma_\tau \rho_{7,t} + \rho_{8,t} - \rho_{10,t} + \Lambda_y^f y_t^f, \quad (34)$$

where $\rho_{7,t}$, $\rho_{8,t}$ and $\rho_{10,t}$ are Luglange multiplyers on Eqs.(1), (22) and (28), respectively. Eq.(33) is the FONC under the Exact policy and shows that the target level output is the efficient level of output. Eq.(34) is the FONC under the false policy and shows that the target level output is the efficient level of output when there is no default risk.[7]

The FONC for the inflation is given by:

$$\Lambda_\pi \pi_t = -\frac{\tau + \varsigma_\tau \sigma_B}{\varsigma_\tau \sigma_B}\rho_{1,t} + \frac{1}{\beta}\rho_{1,t-1} - (\rho_{2,t} - \rho_{2,t-1}) - \frac{\tau}{\beta\left(\tau + \phi\varsigma_\tau \sigma_B\right)}\rho_{6,t}, \quad (35)$$

where $\rho_{1,t}$ and $\rho_{6,t}$ are Laglange multiplyers on Eqs.(20) and (4), respectively. Because of commitment, lagged Laglange multiplyers appear in Eq.(35). Eq.(35) is common to both the Exact and false policies. In Eqs.(33), (34) and (35), $\rho_{2,t}$ appears and those equalities imply that the inflation

[7] To derive Eqs.(33) and (34), we use th FONCs for the marginal cost and the hours of labor and eliminate lLaglange multiplyers on Eq.(22) $\rho_{8,t}$.

is stabilized through stabilizing output (or, stabilizing welfare relevant output gap or difference between the output and the target level output). This mechanism is similar to other literatures on optimal monetary policy and is common to both the Exact and the false policies.

The FONCs for the government coupon gap are given by:

$$\Lambda_r \hat{r}_t^G = \Lambda_r \hat{r}_t^H - \rho_{5,t} + \frac{\beta (\tau + \varsigma_\tau \sigma_B)}{\varsigma_\tau \sigma_B} \rho_{1,t+1} + \frac{\tau}{\varsigma_\tau \sigma_B} \rho_{6,t}, \quad (36)$$

$$0 = -\rho_{5,t} + \frac{\beta (\tau + \varsigma_\tau \sigma_B)}{\varsigma_\tau \sigma_B} \rho_{1,t+1} + \frac{\tau}{\varsigma_\tau \sigma_B} \rho_{6,t}, \quad (37)$$

where $\rho_{5,t}$ is a Laglange multiplyer on Eq.(3) and Eqs.(36) and (37) are the FONCs under the exact and the false policies, respectively. Eq.(36) implies that the policy authorities have to minimize the premium difference $\hat{r}_t^R \equiv \hat{r}_t^H - \hat{r}_t^G$ to minimize the welfare costs, although Eq.(37) implies that there is no explicit incentive to minimize it.

The FONCs for the government debt yield gap are given by:

$$\Lambda_r \hat{r}_t^H = \Lambda_r \hat{r}_t^G - \rho_{1,t} + \rho_{3,t} - \rho_{4,t}, \quad (38)$$

$$0 = -\rho_{1,t} + \rho_{3,t} - \rho_{4,t}, \quad (39)$$

where $\rho_{4,t}$. Eqs.(38) and (40) are the FONCs under the exact and the false policies, respectively. Eq.(38) implies that the policy authorities have to minimize the premium difference $\hat{r}_t^R \equiv \hat{r}_t^H - \hat{r}_t^G$ to minimize the welfare costs, although Eq.(40) implies that there is no explicit incentive to minimize it.

4 Numerical Analysis

4.1 Parameterization

We run a series of dynamic simulations and adopt the following benchmark parameterization. The calibrated parameters are shown in Tab.1.[8] In addition, we assume that the productivity a_t and government expenditure g_t follow AR(1) processes and that persistency is 0.9. As shown in Tab. 1,

4.1.1 Impulse Response Functions

We discuss the impulse response functions (IRFs). Figs. 2 and 3 show IRFs under low interest spread ($\phi = 0.00267$) in the steady state to a one percent decrease in the productivity and on a percent increase on the government expenditure, respectively. In those figures, there are few differenecs on IRFs between the exact and the false. Both the inflation and the welfare relevant output gap are well stabilized under the false while tose are somewhat fluctuating under the exact (Panels 1 and 4 in Figs. 2 and 3). In addition, the premium gap under the false is more fluctuating than it under the exact. This stems from differenes on welfare costs function and it is very easy to understand why. Under the false policy, inflation is stabilized agressively because of high weight of the quadratic term of inflation on the welfare costs function. On the other hand, there is no weight on the quadratic term of the premium difference.

[8] Creedy and Gremmell[?] report that the tax revenue elasticity ranges from 0.5 to 1 and we choose 1 as the tax revenue elasticity ς_τ.

Next, we discuss on IRFs under high interest spread in the steady state ($\phi = 0.138$) and see Figs. 4 and 5. First, we discuss on IRFs to a one percent decrease in the productivity. A decrease in the productivity decreases the target level output under both policies as shown in the definition of the target level output. Although target level output depends on the staedy state value of the interest spread under the exact policy, difference on the target level output can be ignored (Panel 3). Under the false policy, the tax gap fall enough although the tax gap does not fall enough to boost output under the exact policy (Panel 9). As a result, the output and the welfare relevant output gap under the exact policy falls more under the exact policy while the output approximateli corresponds to its target level and the welfare relevant output gap is close to zero, under the false policy (Panels 1 and 2). Through stabilizing the welfare relevant output gap, the infaltion is stabilized under the false policy while this is not, under the exact policy.

On the one hand, drastic decrease in the tax gap worsens the fiscal surplus under the false policy although the fiscal surplus is not so worsend under the exact policy because the tax gap is not so lowered. Here, the fiscal surplus under the exact policy which does not so fall in contrast with it under the false policy has important role to stabilize the premium gap which is:

$$\begin{aligned}\hat{r}_t^R &\equiv \hat{r}_t^H - \hat{r}_t^G \\ &= \hat{r}_t^H - \hat{r}_t - \left(\hat{r}_t^G - \hat{r}_t\right) \\ &= \hat{r}_t^S - (-\phi sp_t) \\ &= \frac{\phi \varsigma_\tau \sigma_B}{\tau} \mathrm{E}_t \left(\hat{\delta}_{t+1} - \delta_{t+1}^*\right)\end{aligned}$$

As shown in the second and the third lines in the previous expression, the smaller the fiscal surplus, the smaller the government coupon rate premium $\hat{r}_t^G - \hat{r}_t$. Then, the government coupon gap approximately comove with the nominal interest gap. In addition, because the infaltion is not so stabilized, the government debt decreases more under the exact policy, as shown in Eq.(4) (Panel 10). As shown in Eq.(15), a decrease in the government debt decreases the government debt yield. Meanwhile, the nominal interest gap decreases (Panel 8). Then, the interest spread for risky assets $\hat{r}_t^S$ does not so widen but converges. As shown in the second line in the previous expression, small interest spread for risky assets and small government coupon premium reduces the premium gap. In fact, the premium difference is more aggressively stabilized under the exact policy although it rises considerably under the false policy (Panel 5). As shown in line 4 in the previous experssion, the smaller the the premium difference, the smaller the deviation of the expected default rate from its target level. Reflecting this fact, the default gap is well stabilized under the exact policy although the default gap does not converges immediately under the false policy (Panel 6).

The default gap under the exact policy which rises sharply after the shock is consistent with Uribe[27]'s result. Uribe analyze one of monetary policy, 'interst rate peg' policy which pegs the nominal interest rate for risky asstes to it for safty assets under an economy with sovereign risk. His interest paeg policy rises the default rate sharply after an exogenous negative fiscal surplus shock although a rise in the default rate ties up immediately. His interest paeg policy corresponds to the policy minimizing the interest spread for risky assets $\hat{r}_t^S$ in our paper. As Eq.(14) implies, the policy minimizing the interest spread for risky assets is equivalent to the policy minimizing to the expected default gap $\mathrm{E}_t\left(\hat{\delta}_{t+1}\right)$. If the policy authorities adopt the policy minimizing the interest spread for risky assets and they succeeds to conduct it, the expected default rate becomes zero. Thus, although a rise in the default gap immediately after the shock is innevitable, the default gap becomes zero after that. Our exact policy has a feature minimizing the default gap it self as the previous expression implies and the default gap rises afeter the shock sharply and converges immediately although it takes time to fully converges. This is a reason why our result is consistent with Uribe[27]'s result.

Next, we discuss on the IRFs to one percent increase in the govenment expenditure. An increase in the government expenditure increases the target level output and applies to infaltion pressure (Panel 3). To stabilize inflation, the tax gap is hiked under both the exact and false policies and the fiscal surplus improves although it worsens for the moment. However, improving on the fiscal surplus under the exact policy is faster than it under the false policy (Panel 5). Faster improving on the fiscal surplus stabilizes the premium difference under the exact policy although the premium difference fluctuates under the false policy (Panel 4). Because of stabilized premium difference, the default gap is immediately stabilized under the exact policy although fluctuation of the default gap is lasting under the false policy (Panel 4).

Notice that fluctuation of the default gap under the false policy is not sosevere and is relatively close to it under the exact policy. The reason why the fluctuation of the default gap is not sosevere is that the interest spread for the risky assets r_t^S is not so different between the exact and the false policies. As shown in Eq.(15), the interest spread for the risky assets depends on the government debt and the fluctuation of the government debt under the false policy is close to it under the exasct policy. Thus, the premium difference $\hat{r}_t^R$ under the false policy is not so severe (Remember that the premium difference consists of the coupon premium $\hat{r}_t^G - \hat{r}_t$ and the interest spread for the risky assets r_t^S). Thus, the default gap under the false policy is not so severe under the false policy. For reference, we note that the standard deviation of the default gap to one percent increase in the government expenditure under the exact policy is 0.5946 and it under the false policy is 0.6140.

4.2 Optimal Monetary and Fiscal Policy Rules

This section introduces the simple policy rules. The rule for the central bank takes a class of Taylor rule:

$$\hat{r}_t = \varphi_\pi \pi_t + \varphi_x x_t, \tag{40}$$

where $x_t \equiv y_t - \bar{y}_t$ denotes the output gap, namely, difference between output and its natural level and $\bar{y}_t = -\frac{\sigma_C \tau}{(1+\sigma_C \psi)(1-\tau)}\hat{\tau}_t + \frac{\sigma_G}{1+\sigma_C \psi} g_t + \frac{(1+\psi)\sigma_C}{1+\sigma_C \psi} a_t$ denotes the natural level of output. The natural level of output can be derived by Plugging Eqs.(22) and (28) into Eq.(26) and imposing $mc_t = 0$ which implies that the marginal cost is constant for all t.[9]

Another rule for the government conducting fiscal policy takes a form:

$$\hat{\tau}_t = -y_t + \frac{\varphi_b}{\varsigma_\tau} b_{t-1} + \frac{\sigma_G}{\tau} g_t, \tag{41}$$

which can be rewritten as:

$$sp_t = \varphi_b b_{t-1}.$$

by using Eq.(1). Thus, Eq.(41) is a class of Bohn rule.

We find both φ_π and φ_b through grid search. As hown in Tab. 2, there are no distiction between the exact and the false. The both of φ_π and φ_b are same under both policies. This implies that the policy authorities do not need to aware of the default risk. Whether they are aware of it

[9]We derive the natural level of output following Gali and Monacelli[?].

or not, there is one policy rule and there are no differences on policy rules. However, under high interest spread in the steady state, the policy rules are different between two policies, as shown in Tab. 2. Comparing the rule under high interest spread in the steady state with it under low interest spread in the steady, we find that easier policy is adpted under high interets spread in the steady state. This means that the cost derived by fluctuation of the premium difference is no longer negligible if the staedy state value of the interest spread is high. If the staedy state value of the interest spread is high, policy authorities should give up to stabilize inflation and agressively stabilize the premium difference.

5 Welfare Analysis and Optimal Monetary and Fiscal Policy Rules

5.1 Welfare Analysis

Now we analyze welfare brought about by both policies. First of all, we define our welfare criteria. Setting $\beta \to 1$ on $\sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left(L_t\right)$, the expected welfare costs is given as follows:

$$\sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left(L_t\right) = \frac{\Lambda_y}{2} \mathrm{var}\left(y_t - y_t^*\right) + \frac{\Lambda_\pi}{2} \mathrm{var}\left(\pi_t\right) + \frac{\Lambda_r}{2} \mathrm{var}\left(\hat{r}_t^R\right),$$

which shows that our welfare costs criteria consists of weighted sum of variances on the welfare relevant output gap, the inflation and the coupon rate gap. As shown in Tab. 3, welfare costs are not so different under low interest spread in the steady state. Especially, welfare costs are same under the rule between the exact and the false beacuse there are no distictions on policy rules under the both policies if the interest spread in the steady state is low. However, under high interest spread, the differences on the welfare costs are not negligibe. Under the false, welfare costs are 10.4 times larger than those under the exact. Thus, it is very important for policy authrotities to be aware the default risk if the interst spread is enough high.

6 Conclusion

We develop a class of DSGE moldes with nominal rigidities and we introduce the default risk in the model. We find that policy rules are same if the interest spread in the steady stae is low whether policy authorities are aware of the default risk or not. However, if the steady state value of interest spread is high, the policy rule is different. If policy authorities are not aware of the default risk, the policy authoroties introduce tighter policy which generates much welfare costs.

Appendices

A Nonstochastic Steady State

We focus on equilibria where the state variables follow paths that are close to a deterministic stationary equilibrium, in which $\Pi_t = 1$ and $\frac{\tilde{P}_t}{P_t} = 1$. Because this steady state is nonstochastic, the productivity has unit values; i.e., $A = 1$.

In this steady state, the gross nominal interest rate is equal to the inverse of the subjective discount factor, as follows:

$$R = \beta^{-1}.$$

Because of $\Gamma(0) = 1$, The definition of the government debt coupon rate boils down to:

$$R^G = R.$$

Notice that $sp_t = 0$ in the steady state.

Eq.(23) can be rewritten as:

$$\frac{\tilde{P}_t}{P_t} = \mathrm{E}_t\left(\frac{K_t}{F_t}\right) \tag{A.1}$$

with:

$$K_t \equiv \frac{\varepsilon}{\varepsilon - 1}\sum_{k=0}^{\infty}\left(P_{t+k}C_{t+k}\right)^{-1}\tilde{Y}_{t+k}MC_{t+k}^n \quad ; \quad F_t \equiv P_t\sum_{k=0}^{\infty}\left(P_{t+k}C_{t+k}\right)^{-1}\tilde{Y}_{t+k},$$

which are bolis down to in the steady state:

$$K = \frac{\frac{\varepsilon}{\varepsilon-1}YMC^n}{(1-\alpha\beta)(PC)} \quad ; \quad F = \frac{PY}{(1-\alpha\beta)(PC)}.$$

Plugging those equalities in the steady state condition of Eq.(A.1), namely, $K = F$ yields:

$$P = \frac{\varepsilon}{\varepsilon - 1}MC^n,$$

which can be rewritten as:

$$MC = \left(\frac{\varepsilon}{\varepsilon - 1}\right)^{-1}. \tag{A.2}$$

Furthermore, Eqs.(25) and (A.2) imply the following:

$$\frac{U_N}{U_C} = \frac{1-\tau}{\left(\frac{\varepsilon}{\varepsilon-1}\right)\mu^w} = 1 - \Phi,$$

with $U_C = C^{-1}$ and $U_N = N^{\psi}$. Note that because $\tau \in (0,1)$ and $\varepsilon > 1$, this steady state is distorted.

The definition of R_t^H boils down to in the steady state as follows:

$$R^S = \left[1 + \frac{B}{SP}\Gamma'(0)\right]. \tag{A.3}$$

Eq.(13) boils down to in the steady state as follows:

$$R^S = (1-\delta)^{-1} \tag{A.4}$$

Plugging Eq.(A.4) into Eq.(A.3) and rearranging, we have:

$$\delta = \frac{\phi\varsigma_\tau\sigma_B}{\tau + \phi\varsigma_\tau\sigma_B},$$

where we use $\frac{B}{SP} = \left(\frac{SP}{Y}\right)^{-1}\frac{B}{Y}$.

References

[1] Andrews, D.W.K. (1991), "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation," *Econometric*, 50, 1029–1054.

[2] Arellano, C. (2008), "Default Risk and Income Fluctuations in Emerging Economies," *American Economic Review*, 98, 690–712.

[3] Bacchetta, P, Perazzi, E. and Wincoop, E (2015), "Self-Fulfilling Debt Crises; Can Monetary Policy Really Help?" Unpublished Manuscript.

[4] Benigno, P. (2001), "Price Stability with Imperfect Financial Integration," Unpublished Manuscript, New York University.

[5] Benigno, P. and Benigno, G. (2006), "Designing Targeting Rules for International Monetary Policy Cooperation," *Journal of Monetary Economics*, 53, 473–506.

[6] Benigno, P. (2009), "Price Stability with Imperfect Financial Integration," *Journal of Money, Credit and Banking*, 41, 121–149.

[7] Benigno, P. and Lopez-Salido, D. (2006), "Inflation Persistence and Optimal Monetary Policy in the Euro Area," *Journal of Money Credit and Banking*, 38, 587–614.

[8] Benigno, P. and Woodford, M. (2005), "Inflation Stabilization and Welfare: The Case of a Distorted Steady State," *Journal of the European Economics Association*, 3, 1185–1236.

[9] Benigno, P. and Woodford, M. (2003), "Optimal Monetary and Fiscal Policy: A Linear-Quadratic Approach," *NBER Macroeconomics Annual*, 18, 271–333.

[10] Cochrane, J.H. (1998), "A Frictionless View of U.S. Inflation," *NBER Macroeconomics Annual*, 13, 323–384.

[11] Corsetti, G. and Dedola, L. (2016), "The Mystery of the Printing Press: Monetary Policy and Self-fulfilling Debt Crises," *Journal of the European Economic Association*, 14, 1329–1371.

[12] Corsetti, G., Kuester, K., Meier, A. and Mueller, G.J. (2013), "Sovereign Risk, Fiscal Policy, and Macroeconomic Stability," *Economic Journal*, 123, F99–F132.

[13] Curdia, V. and Woodford, M. (2009), "Credit Frictions and Optimal Monetary Policy," *BIS Working Papers*, #278, Bank for International Settlements.

[14] Erceg, Christopher J., Dale W. Henderson and Andrew T. Levin (2000), "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of Monetary Economics*, 46, 281–313.

[15] Ferrero, A. (2009), "Fiscal and Monetary Rules for a Currency Union," *Journal of International Economics*, 77, 1–10.

[16] Gali, Jordi (2008), *Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework*, Princeton University Press, Princeton, NJ.

[17] Gali, J. and Gertler, M. (1999), "Inflation Dynamics: A Structural Econometric Analysis," *Journal of Monetary Economics*, 44, 195–222.

[18] Gali, J. and Monacelli, T. (2005), “Monetary Policy and Exchange Rate Volatility in a Small Open Economy,” *Review of Economic Studies*, 72, 707–734.

[19] Creedy, J. and Gremmell, N. (2010), “The Elasticity of Taxable Income and the Tax Revenue Elasticity,” *Department of Economics*, *The University of Melbourne Working Paper Series*, No. 1110.

[20] Leeper, E.M. (1991), “Equilibria under ‘Active’ and ‘Passive’ Monetary and Fiscal Policyies, ” *Journal of Monetary Economics*, 27, 129–147.

[21] Mendoza, E.E. and Yue, V.Z. (2012), “A General Equilibrium Model of Sovereign Default and Business Cycles,” *Qurterly Journal of Economics*, 127, 889–946.

[22] Okano, Eiji (2014), “How Important is Fiscal Policy Cooperation in a Currency Union?” *Journal of Economic Dynamics and Control*, 38, 266–286.

[23] Okano, Eiji and Kazuyuki Inagaki (2017), “Revisiting the Fiscal Theory of Sovereign Risk from a DSGE Viewpoint,” *The Society of Economics, Nagoya City University Discussion Paper Series* No. 611.

[24] Okano, Eiji and Masataka Eguchi (2017), “A Role Model of Monetary and Fiscal Policy Rules in an Economy with Sovereign Risk: Evidence from Indonesia,” Mimeo.

[25] Schabert, Andreas (2010), ’Monetary Policy under A Fiscal Theory of Sovereign Default,” *Journal of Economic Theory*, 145, 860–868.

[26] Tesar, L.L. (1995), ”Evaluating the Gains from International Risk Sharing,” *Carnegie–Rochester Conference Series on Public Policy*, 42, 95–143.

[27] Uribe, M. (2006), “A Fiscal Theory of Sovereign Risk,” *Journal of Monetary Economics* 53, 1857–1875.

[28] Woodford, M. (2003), “Interest Rate and Prices: Foundations of a Theory of Monetary Policy,” *Princeton University Press*, Princeton and Oxford.

[29] Woodford, M. (1994), “Monetary Policy and Price Level Determinacy in a Cash-in-advance Economy,” *Economic Journal*, 4, 345–380.

Table 1: Parameterization

Parameter	Value	Source
β	0.99	Gali and Monacelli[18]
ψ	3	Gali and Monacelli[18]
θ	0.66	Gali and Monacelli[18]
ε	6	Gali and Monacelli[18]
ϕ	0.00267	Long-term Government bond yields (10-year) and Effective FF rate in the US
	0.138	Okano and Inagaki[23]
γ	1.145	Okano and Inagaki[23]
τ	0.3	Ferrero[15]
σ_G	0.276	Ferrero[15]
σ_B	2.4	Ferrero[15]
ς_τ	1	Creedy and Gremmell[19]

Table 2: Coefficients on Rules

Coefficients	Low		High	
Interest Spread	φ_π	φ_b	φ_π	φ_b
Exact	4.4889	1.2	14.0556	0.5333
False	4.4889	1.2	23.6222	1.2

Table 3: Welfare Gains from Exact Policy

Interest Spread	Low	High
Optimal	30.93%	90.42%
Rule	0%	29.48%

Figure 1: Feature of Welfare Costs Function

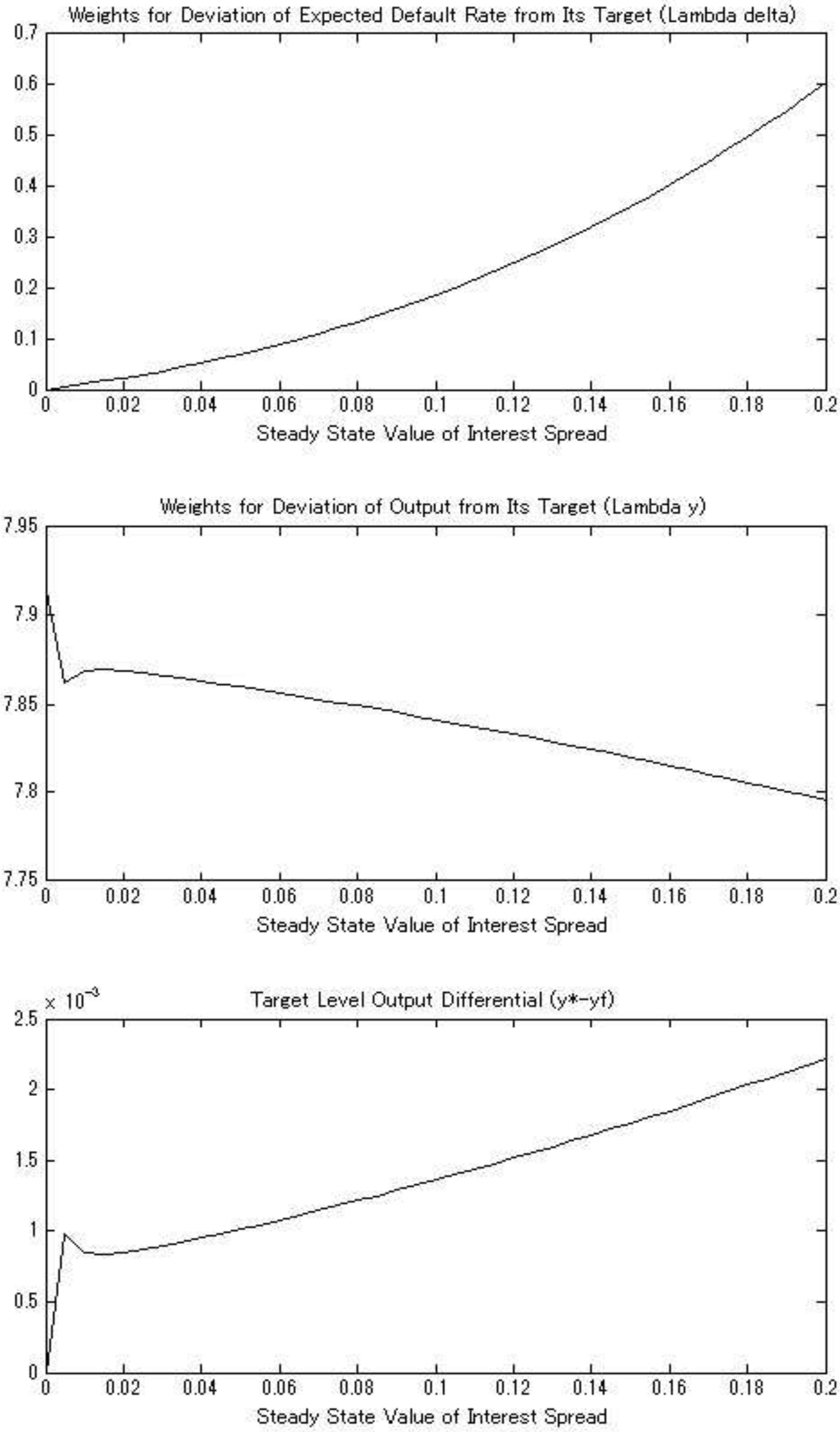

Figure 2: IRFs to Unit Decrease in the Productivity under Low Interest Spread

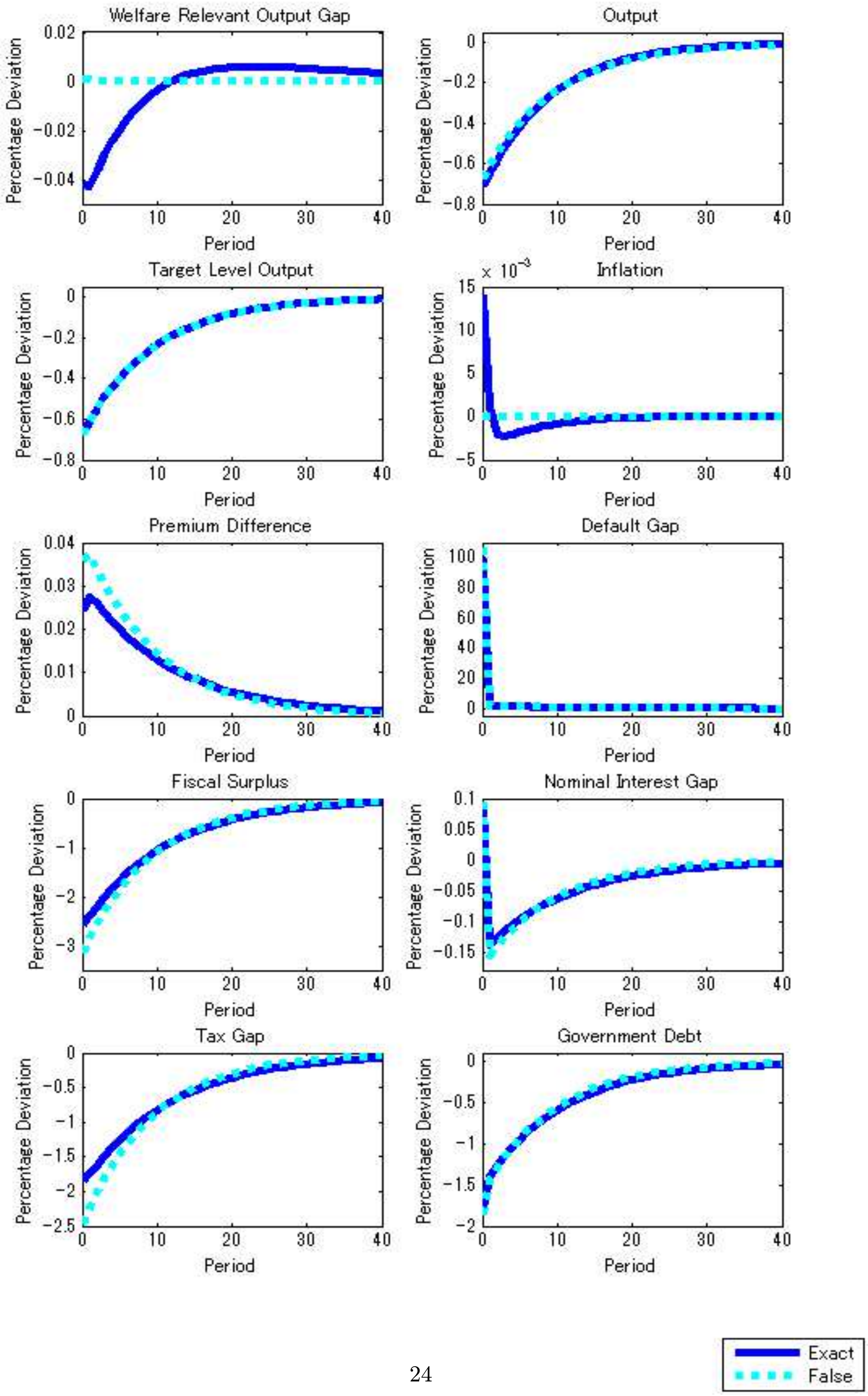

Figure 3: IRFs to Unit Increase in the Government Expenditure under Low Interest Spread

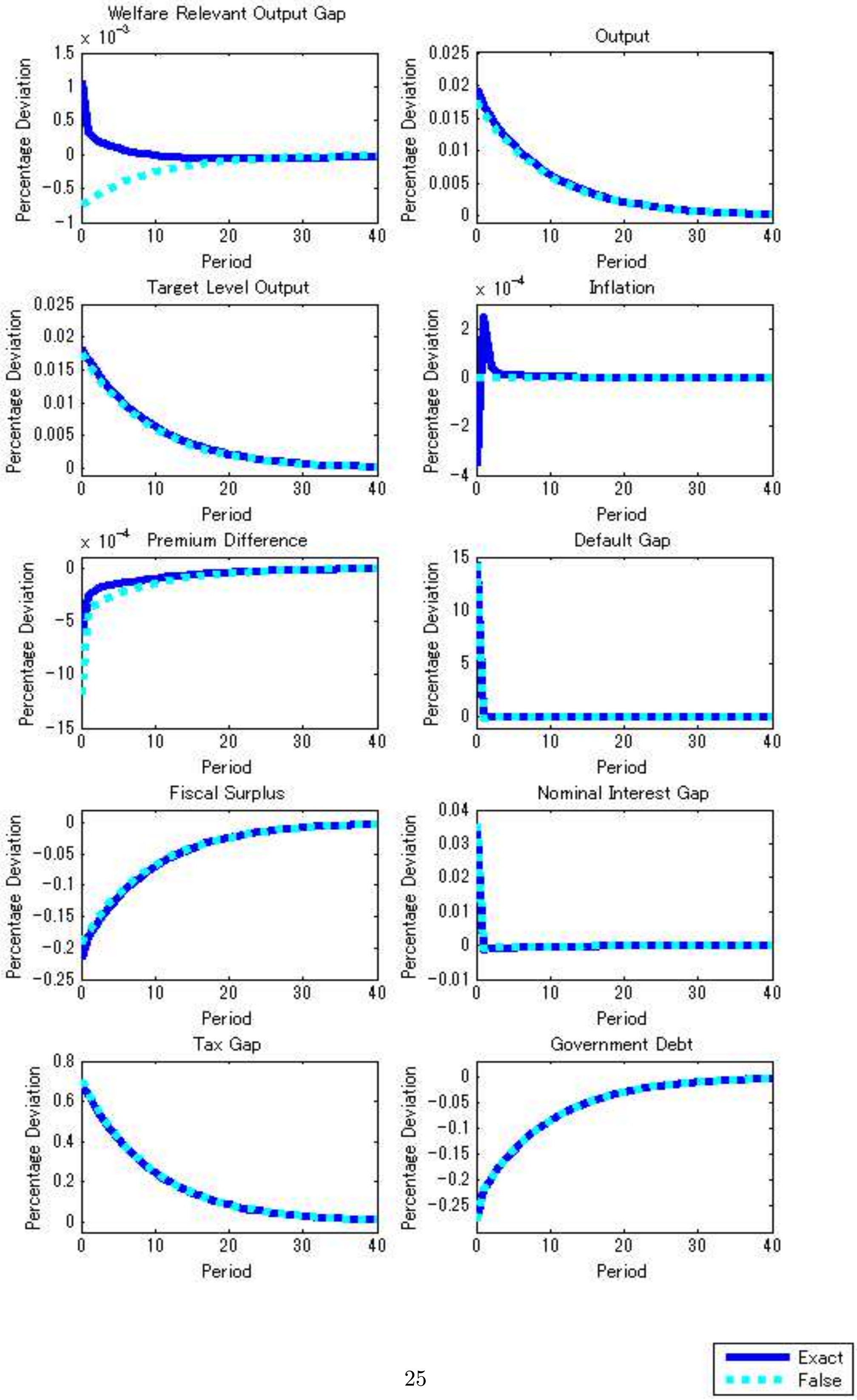

Figure 4: IRFs to Unit Decrease in the Productivity under High Interest Spread

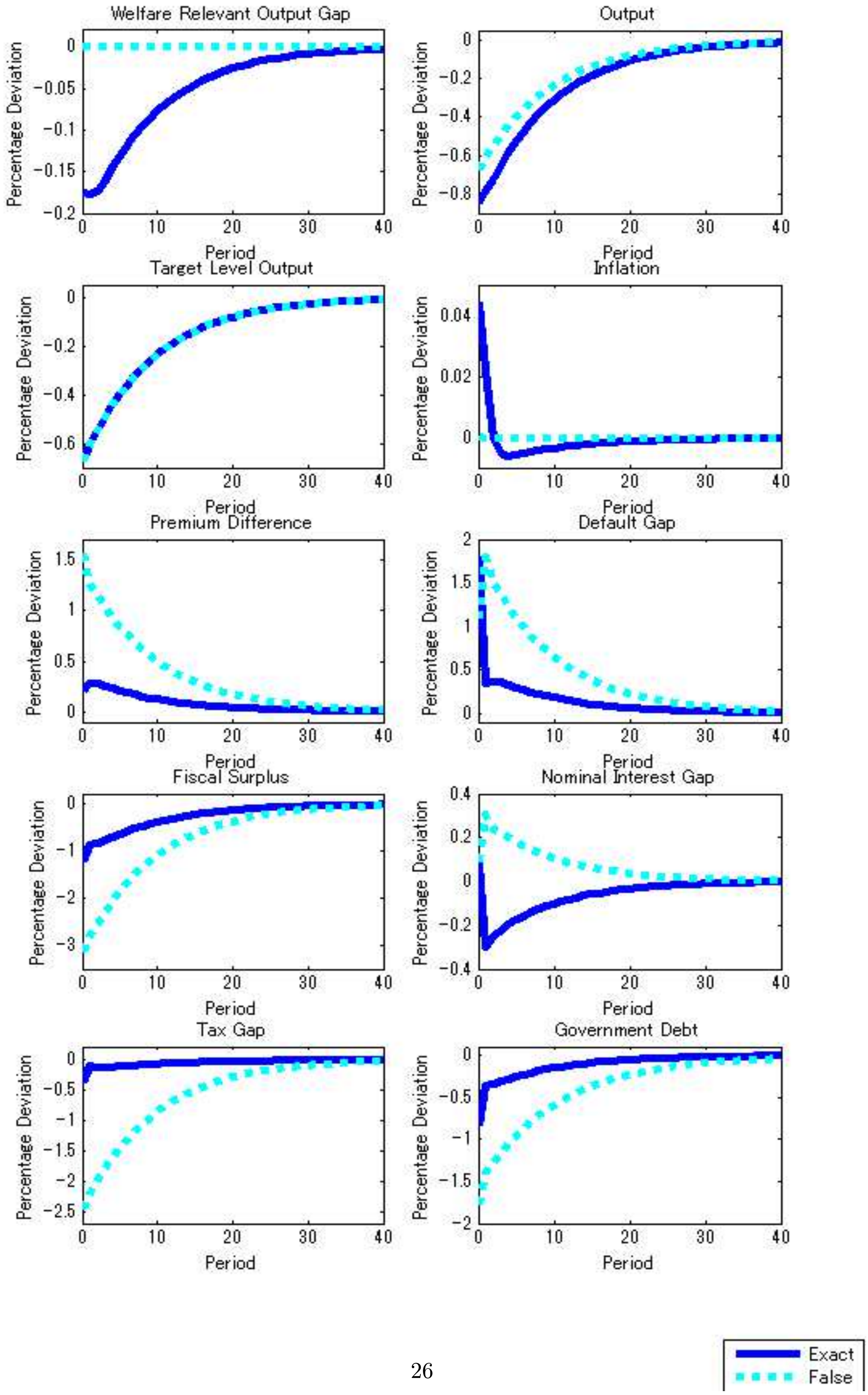

Figure 5: IRFs to Unit Increase in the Government Expenditure under High Interest Spread

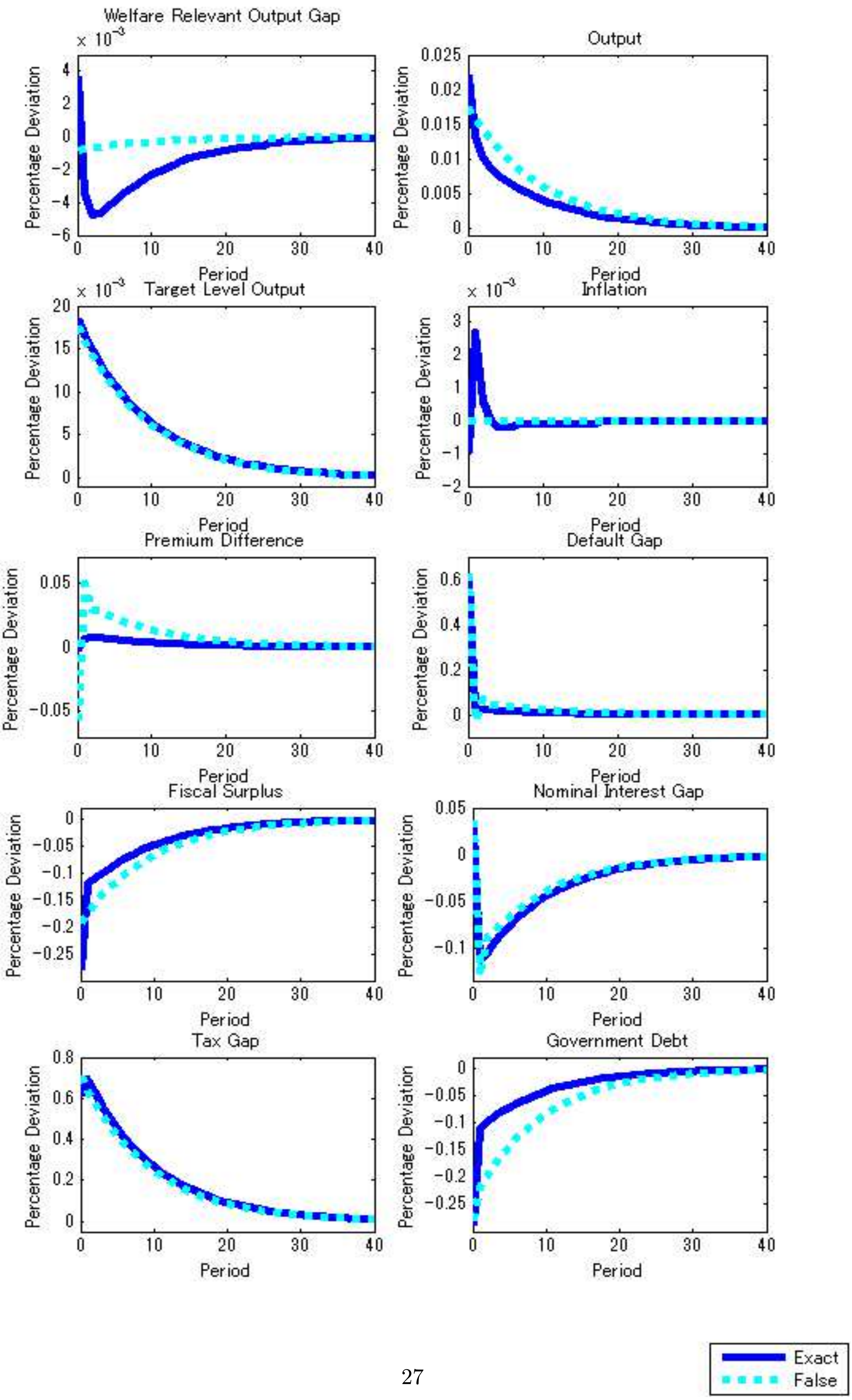

Figure 6: IRFs under the Exact Policy

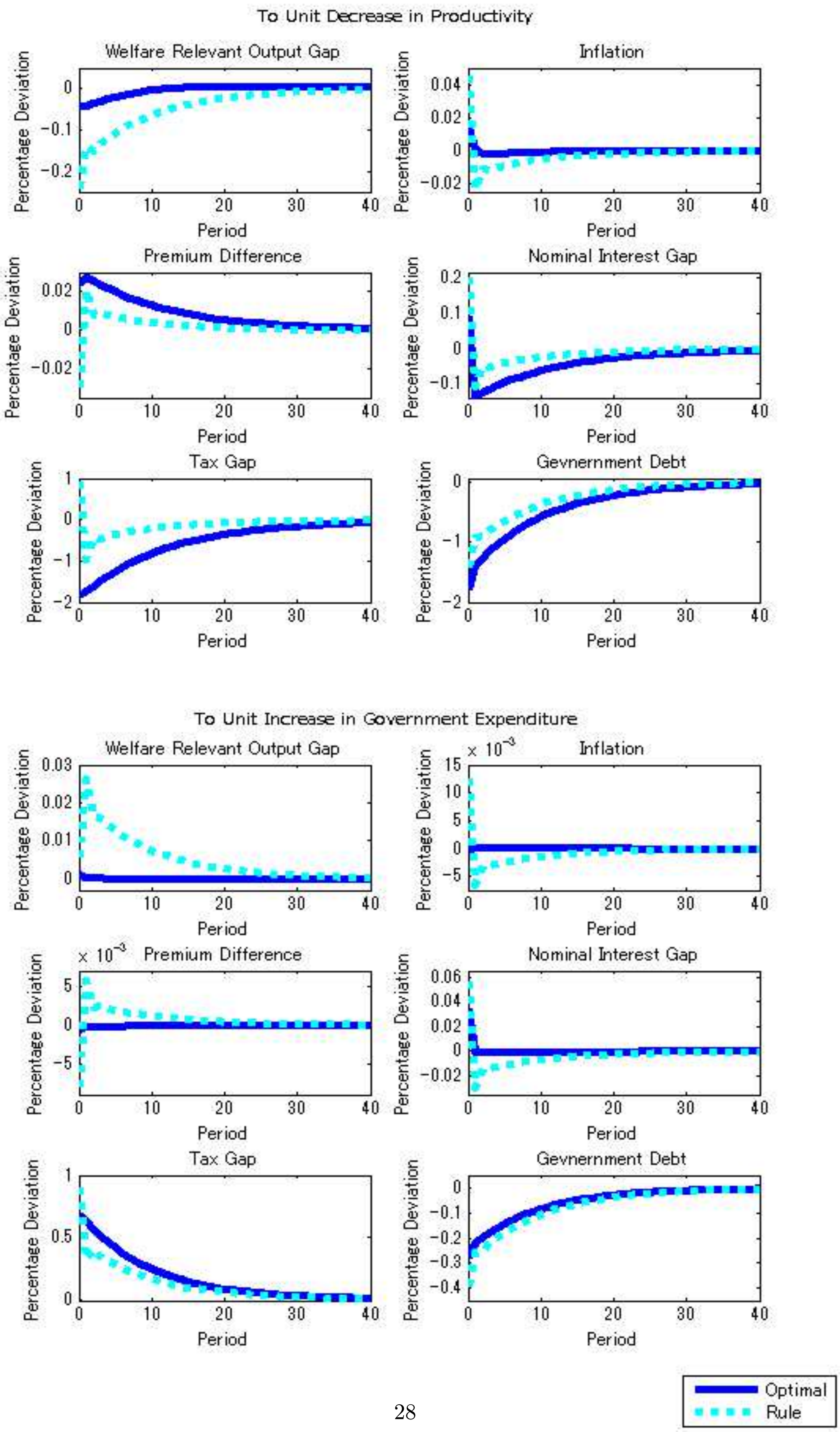

Figure 7: IRFs under the False Policy